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Thus, $[(e_1 e_2 \ldots\ldots e_m)(e_{m+1} \ldots\ldots e_n)] = [e_1 e_2 \ldots\ldots e_n]$.

53. *In order to multiply two simple quantities,* $[ab \ldots\ldots]$ *and* $[cd \ldots\ldots]$, *it is sufficient to unite their simple factors taken in order into a single combinatory product* $[ab \ldots\ldots cd \ldots\ldots]$.

Proof.—Let $e_1 \ldots\ldots e_n$ be the original units, and let $a = \Sigma \alpha_a e_a$, $b = \Sigma \beta_b e_b$, $c = \Sigma \gamma_c e_c$, $d = \Sigma \delta_d e_d$. Then

$$[(ab \ldots\ldots)(cd \ldots\ldots)] \equiv [(\Sigma \alpha_a e_a . \Sigma \beta_b e_b \ldots\ldots)(\Sigma \gamma_c e_c . \Sigma \delta_d e_d \ldots\ldots)]$$

$$= [\Sigma \{\alpha_a \beta_b \ldots\ldots [e_a e_b]\} \Sigma \{\gamma_c \delta_d \ldots\ldots [e_c e_d]\}] \ldots\ldots\ldots\ldots (28)$$

$$= \Sigma \{\alpha_a \beta_b \ldots\ldots \gamma_c \delta_d \ldots\ldots [(e_a e_b \ldots\ldots)(e_c e_d \ldots\ldots)]\} \ldots\ldots (28)$$

$$= \Sigma \{\alpha_a \beta_b \ldots\ldots \gamma_c \delta_d \ldots\ldots [e_a e_b \ldots\ldots e_c e_d \ldots\ldots]\} \ldots\ldots\ldots (52)$$

$$= [\Sigma \alpha_a e_a . \Sigma \beta_b e_b \ldots\ldots \Sigma \gamma_c e_c . \Sigma \delta_d e_d \ldots\ldots] \ldots\ldots\ldots\ldots (28)$$

$$\equiv [ab \ldots\ldots cd \ldots\ldots].$$

54. Corollary to 53.—If a simple quantity A is subordinate to B (18), then B may be written $B = [AC]$, where C is a simple factor.

55. *To show that* $[A(BC)] = [ABC]$, *i. e. to show that the associative law holds.*

Proof.—1. When A, B, and C are the products of simple factors, the truth of this case follows readily from 53.

2. When A, B, and C are sums of simple quantities, $A = \Sigma A_a$, $B = \Sigma B_b$, $C = \Sigma C_c$.

$$[(A(BC)] \equiv [\Sigma A_a . (\Sigma B_b \Sigma C_c)] = \Sigma [A_a (B_b C_c)] \ldots\ldots\ldots\ldots (28)$$

$$= \Sigma [A_a B_b C_c] \text{ (By 1, above) } = [\Sigma A_a . \Sigma B_b . \Sigma C_c] \text{ (28) } = [ABC].$$

[To be Continued.]

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A SOLUTION OF THE OBLIQUE TRIANGLE GIVEN TWO SIDES AND THE INCLUDED ANGLE.

By H. C. WHITAKER, Ph. D., Professor of Mathematics, Manual Training School, Philadelphia, Pa.

Take the data to be b, c, A.

$$p(\cot A + \cot B) = c.$$

Divide by $p = b \sin A$ and transpose $\cot A$.

$$\cot B=\frac{c}{b\sin A}-\cot A\,;\; a=\frac{b\sin A}{\sin B}.$$

Find logarithm of $b\sin A$ in the column for a; then in the column for B, find $\frac{c}{b\sin A}$ as a logarithm, then as a number; to find B, use the table of natural cotangents.

If a solution wholly logarithmic is desired, take

$$\tan\theta=\frac{b\sin A}{c}\,;\; \tan B=\frac{\sin A\sin\theta}{\sin(A-\theta)};\; a=\frac{b\sin A}{\sin B}.$$

INTEGRATION OF ELLIPTIC INTEGRALS.

By GEORGE B. McCLELLAN ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High Schoool, Chester, Pa.

[Continued from November Number.]

Since all the expressions in the form of

$$\int_0^{2\pi}\sin^a\theta\cos^{2b+1}\theta(1-e^2\sin^2\theta)^c d\theta=0,$$

(where a, b, c are integers), we will not consider them in our subsequent treatment. From (13_0) we have,

$$\int_0^{\frac{1}{2}\pi}\frac{\sin^2\theta d\theta}{\sqrt{(1-e^2\sin^2\theta)}}=\frac{1}{e^2}[F(e,\,\tfrac{1}{2}\pi)-E(e,\,\tfrac{1}{2}\pi)]\text{............}(3)$$

$$\int_0^{\frac{1}{2}\pi}\frac{\cos^2\theta d\theta}{\sqrt{(1-e^2\sin^2\theta)}}=\int_0^{\frac{1}{2}\pi}\frac{d\theta}{\sqrt{(1-e^2\sin^2\theta)}}-\int_0^{\frac{1}{2}\pi}\frac{\sin^2\theta d\theta}{\sqrt{(1-e^2\sin^2\theta)}}.$$

$$\therefore\int_0^{\frac{1}{2}\pi}\frac{\cos^2\theta d\theta}{\sqrt{(1-e^2\sin^2\theta)}}=\frac{1}{e^2}[E(e,\,\tfrac{1}{2}\pi)-(1-e^2)F(e,\,\tfrac{1}{2}\pi)]\text{..........}(4).$$

If $n=0$ in (10_0), $A_2=\frac{2}{3}[(1+e^2)/e]A_1-\frac{1}{3}A_0$.

$$\therefore A_2=\frac{1}{\pi}\int_0^{2\pi}\frac{\cos 2\varphi d\varphi}{(1+e^2-2e\cos\theta)^{\frac{1}{2}}}$$